Implementation and Comparison of Simulation Methods of Three-Dimensional Scalar Diffraction Fields

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***Abstract*—In the simulation of three-dimensional (3D) optical field distribution, the flexibility in scaling the space of the 3D filed distribution provides a great amount of detailed information of the 3D field. However, the traditional diffraction formulas such as Fresnel diffraction and angular spectrum methods limit the spatial volume of the fields in numerical simulation. In this paper, we present the modification of the 2D diffraction formulas into 3D versions, and the simulation results which were obtained using the chirp z-transform to flexibly scale the spatial volume of the 3D fields. The results can be applied to produce many applications such as 3D imaging and optical clamp.**

***Index Terms*—Fresnel diffraction, Angular spectrum, Chirp z-transform, 3D imaging**

I. Introduction

Many image systems or, such as monitor, projector, digital camera and DV, are two-dimensional image systems. In recent year, developments of 3D movie, TV screen, and camera are prosperous. However, although these technologies give senses that the object is solid and depth, they actually only provide single field of view. We conducted the simulation of 3D distribution of diffraction field. As long as the distribution of diffraction field is simulated correctly, we can then use it to simulate complex incident components. Eventually, we create three-dimensional diffraction field. In the paper, the simulation results of 3D diffraction fields are presented.

II. Theory

Suppose that a plane wave is incident to a small aperture in (ξ,η) plane, and illuminate on an opaque screen in (x,y) plane. There are small fringes around the image on screen, and as the screen moves far away from the aperture, the fringes become more obvious. This phenomenon is called Fresnel diffraction [1-2]. As the screen moves continuously away from the aperture, the variations of fringes become smooth and, finally the fringes stop changing with only the size and shape of image change. This phenomenon is called Fraunhofer diffraction [1-2]. The Fresnel diffraction, and even Fraunhofer diffraction, can be calculated by the formula, given by

$$U(x,y)=\frac{e^{ikz}}{j\lambda z}\iint U(\xi,\eta)\exp\left\{j\frac{k}{2z}\left[(x-\xi)^2+(y-\eta)^2\right]\right\}d\xi d\eta \quad (1)$$

Angular Spectrum considers each Fourier component as plane wave propagating to different direction and uses collection of plane waves to calculate diffraction field on certain distance "z". Angular spectrum also consider phase shift while propagating [1-2] and its representation is

$$U(x,y,z)=\int\int_{-\infty}^{\infty}A\left(\frac{\alpha}{\lambda},\frac{\beta}{\lambda};0\right)\exp\left(j\frac{2\pi}{\lambda}z\sqrt{1-\alpha^2-\beta^2}\right) \times circ\left(\sqrt{\alpha^2+\beta^2}\right)\exp\left[j2\pi\left(\frac{\alpha}{\lambda}x+\frac{\beta}{\lambda}y\right)\right]d\frac{\alpha}{\lambda}d\frac{\beta}{\lambda} \quad (2)$$

Assume a spherical wave goes through a round aperture W, and its incident pupil function is spherical cap. Base on Huygens-Fresnel Principle and method provided by McCutchen [3-4], we can derive a function to calculate the field distribution near focal point. The representation is

$$U_p(x',y',z')\cong-\frac{iA}{\lambda}\frac{f-z'}{f^3}\exp\left[ik\frac{x'^2+y'^2+2z'^2}{2(f-z')}\right]\times \iiint_V U_Q(\xi,\eta,\gamma)P(\xi,\eta,\gamma)\exp\left[-ik\frac{x'\xi+y'\eta+z'\gamma}{f}\right]d\zeta d\eta d\gamma \quad (3)$$

It's called Scaled Three-Dimensional Fourier transform.

III. Simulation

Using these three theories shown in Fig. 1 simulate the distribution of diffraction field; Fig. 1 is based on Angular Spectrum approximation.

In all simulation, in order to observe Fraunhofer diffraction on predictable distant, we assume there is a convergent lens behind the aperture which can make Fraunhofer diffraction appear on the back focal plane. The setting of index are: wavelength(λ = 500 nm), aperture radius(a = 2.5 mm), focal point(FL = 1000 mm), pixel on x direction(dx = 10 μm), pixel on y direction(dy = 10 μm), sample number on x direction(Nx = 1024), sample number on y direction(Ny = 1024). In addition, sample number and pixel on z direction have different setting depending on each theory, and there simulation results are shown on Fig. 2.

Moreover, the results of Fig. 2 after using chirp z-transform [5-6] can achieve more detailed diffraction distribution and the results of using chirp z-transform are shown in Fig. 3

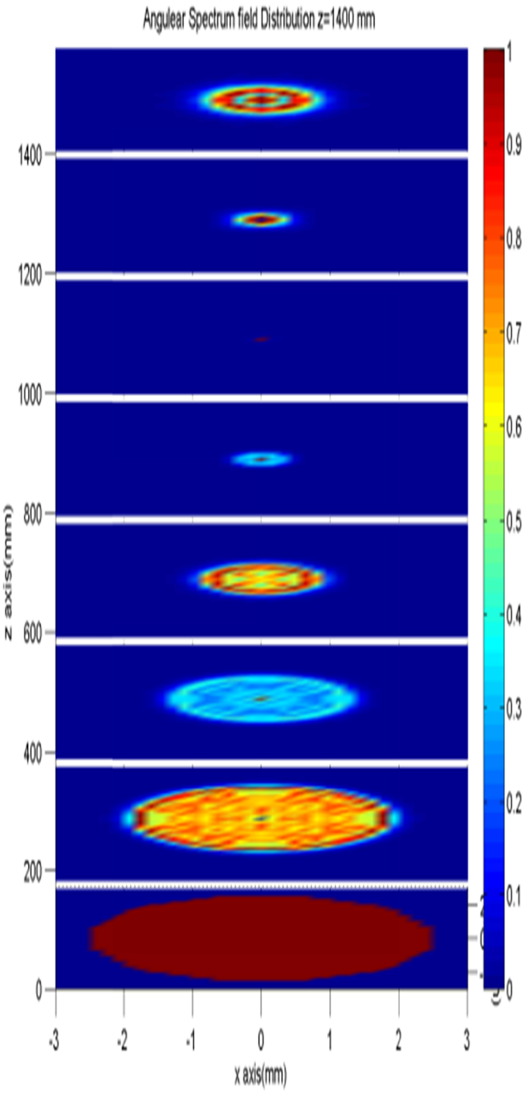
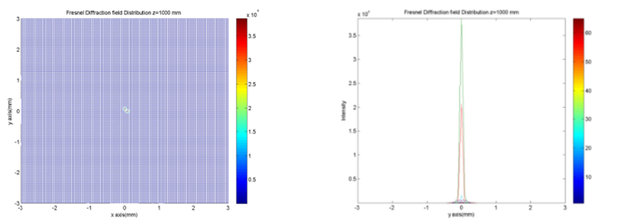
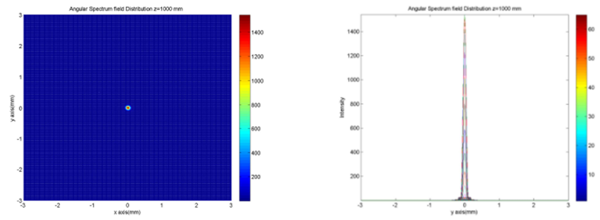


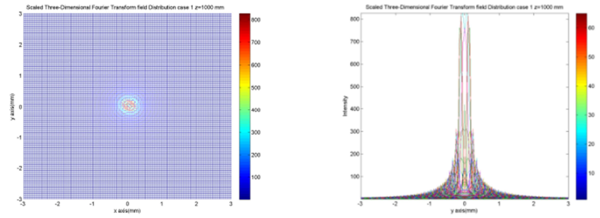
Fig. 1 Angular Spectrum propagating in 3D space



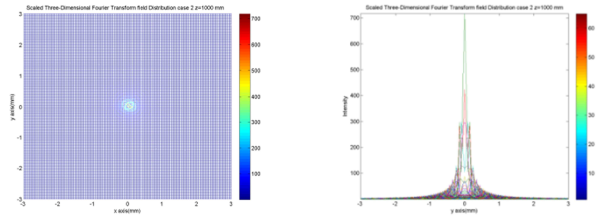
(a) 3D Fresnel diffraction (b) 3D Fresnel diffraction on y-axis



(c) 3D angular spectrum (d) 3D angular spectrum on y direction

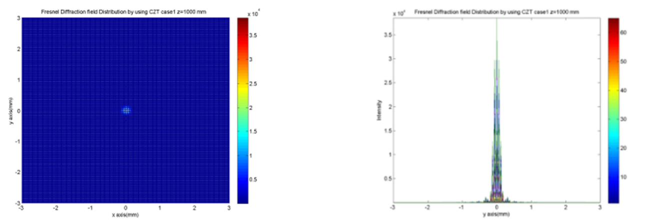


(e) scaled 3D Fourier transform(spherical cap) (f) scaled 3D Fourier transform(spherical cap)on y-axis

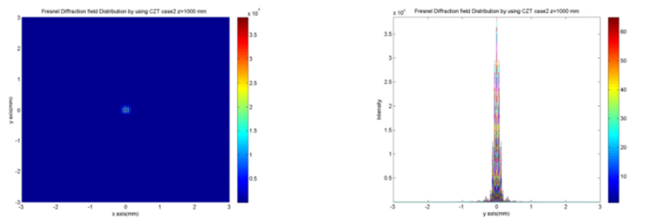


(g) scaled 3D Fourier transform (wavelength) (h) scaled 3D Fourier transform (wavelength)on y-axis

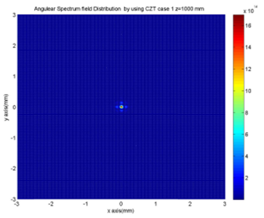
Fig. 2 distribution of diffraction field by using three different theories

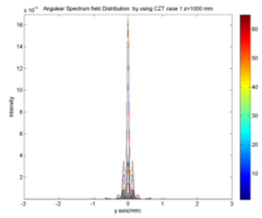


(a) 3D Fresnel diffraction (The sample number is the same as incident field) (b) 3D Fresnel diffraction on y-axis (The sample number is the same as incident field)



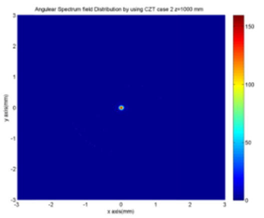
(c) 3D Fresnel diffraction (The sample number isn't the same as incident field) (d) 3D Fresnel diffraction on y-axis (The sample number isn't the same as incident field)

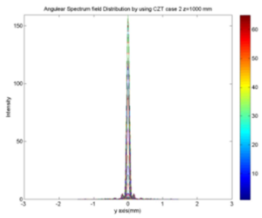




(e) 3D angular spectrum(using FFT ahead, then CZT)

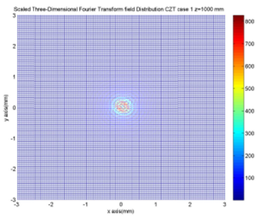
(f) 3D angular spectrum on y direction (using FFT ahead, then CZT)

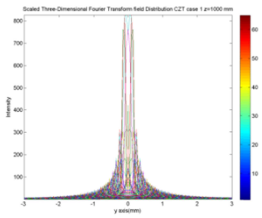




(g) 3D angular spectrum (using CZT ahead, then FFT)

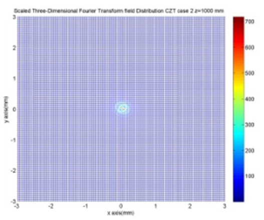
(h) 3D angular spectrum on y direction (using CZT ahead, then FFT)

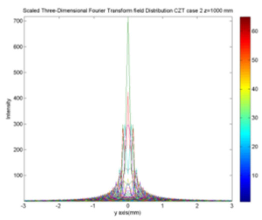




(i) scaled 3D Fourier transform (using the distant of spherical cap as Δz)

(j) scaled 3D Fourier transform on y-axis (using the distant of spherical cap as Δz)





(k) scaled 3D Fourier transform on y-axis (using the ratio of wavelength as Δz)

(l) scaled 3D Fourier transform on y-axis (using the ratio of wavelength as Δz)

Fig. 3 three different theories after using chirp z-transform

IV. CONCLUSION

As shown in Fig. 2, the three theories were all successfully implemented to simulate Fraunhofer diffractions in the focal plane and in the space behind the input component. The results of scaled three-dimensional Fourier transform are more obvious than the two others.

To compare the results of using Fourier transform and the chirp z-transform, the simulation results of these formulas were categorized into two parts as shown in Fig. 3. Because of the use of chirp z-transform, Fraunhofer diffractions become more obvious than before.

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